Periodic Control of Power Electronic Converters

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About the Presenters

Yongheng YANG  Assistant Professor at Aalborg University

He received the B.Eng. degree in electrical engineering and automation from Northwestern Polytechnical University, Shaanxi, China, in 2009 and the Ph.D. degree in electrical engineering from Aalborg University, Aalborg, Denmark, in 2014.

He was a postgraduate student with Southeast University, Jiangsu, China, from 2009 to 2011. In 2013, he was a Visiting Scholar at Texas A&M University, College Station, TX, USA. Since 2014, he has been with the Department of Energy Technology, Aalborg University, where currently he is an Assistant Professor. He has published more than 100 technical papers and coauthored a book *Periodic Control of Power Electronic Converters* (London, UK: IET). His research includes grid integration of renewable energies, power electronic converter design, analysis and control, and reliability in power electronics.

Dr. Yang is a Member of the IEEE Power Electronics Society (PELS) Students and Young Professionals Committee. He served as a Guest Associate Editor of IEEE J. Emerg. Sel. Top. Power Electron. (JESTPE) and a Guest Editor of Applied Sciences. He is an Associate Editor of CPSS Transactions on Power Electronics and Applications.
About the Presenters

Yi TANG  Assistant Professor at Nanyang Technological University

He received the B.Eng. Degree in electrical engineering from Wuhan University, Wuhan, China, in 2007 and the M.Sc. and Ph.D. degrees from the School of Electrical and Electronic Engineering, Nanyang Technological University, Singapore, in 2008 and 2011, respectively.

From 2011 to 2013, he was a Senior Application Engineer with Infineon Technologies Asia Pacific, Singapore. From 2013 to 2015, he was a Postdoctoral Research Fellow with Aalborg University, Aalborg, Denmark. Since March 2015, he has been with Nanyang Technological University, Singapore as an Assistant Professor. He is the Cluster Director in advanced power electronics research program at the Energy Research Institute @ NTU (ERI@N).

Dr. Tang serves as an Associate Editor for the IEEE J. Emerg. Sel. Top. Power Electron. (JESTPE). He Received the Infineon Top Inventor Award in 2012.
Periodic Control of Power Electronic Converters

- **Introduction** (15 mins)
- **Fundamentals in Periodic Control** (45 mins)
- Coffee Break (10 mins)
- **Advanced Periodic Control Schemes** (30 mins)
- **Frequency-Adaptive Periodic Control Strategies** (30 mins)
- Coffee Break (10 mins)
- **Continuing Developments** (30 mins)
- **Summary and Discussions** (10 mins)
Part 1

Fundamentals in Periodic Control
Inaugurated in 1974
22,000 students
2,300 faculty

PBL-Aalborg Model
(Problem-based learning)
Power Electronics Centered

Energy Production | Distribution | Consumption | Control
Focuses at E.T.

E.T. Facts

- **40+** Faculty members
- **100+** Ph.D. students
- **30+** RA and post-docs
- **30+** Visiting scholars and students
- **30+** Technical and administrative staff
- **2** In-house company divisions

60%+ of the above manpower are in power electronics and its applications

2 in-house company divisions heavily involve in power electronics
Power Electronics in today’s power systems:
Power Electronics Dominated power systems:

Revisit or Reinvent the way that electrical energy is processed:

40% Energy Consumption is in electrical energy
60% by 2040

Interfaces
Integration to electric grid
Power transmission, distribution, conversion, control

Power Electronics enable efficient, reliable, flexible conversion and control of electrical energy

Generation...

Consumption...
What is the **Power Electronic** technology:

Refers to **efficient control and conversion** of electrical power by power semiconductor devices.

Side Effect

Power electronic Systems:

- **Topology**
- **Circuit Level**
  - Switching Device
  - Resistor
  - Capacitor
  - Inductor
  - Transformer

- **Component Level**

- **Controller**
  - Control Level
    - Internal feedback
    - Control output
    - Feedforward
    - Feedback
    - Reference

- **Input power** $v_{in}$, $i_{in}$, $f_{in}$
- **Output power** $v_{out}$, $i_{out}$, $f_{out}$
Power electronic conversion brings **Harmonics**: 

![Diagram of power electronic system](image)

- **Input power** $v_{in}$, $i_{in}$, $f_{in}$
- **Output power** $v_{out}$, $i_{out}$, $f_{out}$
- **Topologies**
  - Resistor
  - Capacitor
  - Inductor
  - Transformer
- **Control**
  - Controller
  - Feedforward
  - Feedback
  - Reference
Side Effect

Power electronic conversion brings **Harmonics:**

- Line notching
- Motor vibration
- Overheating
- Triggering resonance
- Equipment dysfunctional
- Nuisance tripping
- ...
π approximation – Liu Hui’s algorithm:
（割圆术，刘徽）
Harmonics are related to **Power Converter Topologies**:
Harmonics are related to **Power Converter Topologies**:

![Diagram showing power converter topologies and grid current](image-url)
Harmonics are related to **Power Converter Topologies**:

- **Grid** with current $i_g$.
- **Converter** topologies with different diode configurations.
- **Rectifier** voltage $v_{rec}$. 

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Tutorial @ IFFECC 2017 – ECCE Asia, Kaohsiung
Harmonics are related to **Power Converter Topologies**:

\[ nk \pm 1 \]  
\[ 6k \pm 1 \]
\[ 12k \pm 1 \]
\[ 24k \pm 1 \]

\( n \)-pulse converters produce dominant \( nk \pm 1 \) \((k = 0, 1, \ldots)\) order harmonics due to \( n \)-pulse commutation
Harmonics due to **Switching and Background Distortions:**

According to KVL:

\[ i_g = \frac{1}{L} \int (v_{ab} - v_g) \, dt = i_g^1 + \frac{1}{L} \sum_{h=2}^{n} \int v_{ab}^h \, dt + \frac{1}{L} \sum_{h=2}^{n} \int (-v_g^h) \, dt \]

\[ i_g \]

Switching

Grid Distortions
Switching harmonic **Injection** and **Compensation**:

Pulse Width Modulation (PWM):

\[
\mathbf{v}_{ab} = d_{\text{pwm}} \mathbf{v}_{dc} = \left( d_{\text{pwm}}^{1} + \sum_{h=2}^{n} \tilde{d}_{h}^{\text{pwm}} \right) \left( \mathbf{v}_{dc} + \tilde{\mathbf{v}}_{dc} \right)
\]
Switching harmonic **Injection and Compensation**:

![Diagram of a power electronic converter](image)

### Pulse Width Modulation (PWM):

\[
v_{ab} = d_{pwm}^1 v_{dc} + d_{pwm}^1 \ddot{v}_{dc} + v_{dc} \sum_{h=2}^{n} \ddot{d}_{pwm}^h + \ddot{v}_{dc} \sum_{h=2}^{n} \ddot{d}_{pwm}^h
\]
Switching harmonic **Injection** and **Compensation**:

Well-designed converter controller ($d_{pwm}$) can remove certain harmonics
Feedback control for **Zero-Error Tracking**:

\[
E(s) = R(s) - Y(s) = \frac{1}{1 + G_c(s)G_p(s)}R(s) - \frac{1}{1 + G_c(s)G_p(s)}D(s)
\]

To achieve zero-error tracking (i.e., \(E(s) \to 0\)):

\[G_c(s) \to \infty\]

- If \(G_c(s) \to \infty\), \(Y(s) \to R(s)\), but system should be **stable**;
- For **periodic signals**, \(G_c(s) \to \infty\) only at desired frequencies is **necessary**.
What is a Periodic Signal:

"A signal is a periodic signal if it completes a pattern within a measurable time frame, called a period and repeats that pattern over identical subsequent periods."

Decomposed into its Fourier Series

https://en.wikibooks.org/wiki/Signals_and_Systems/Periodic_Signals
Harmonic signal generators (Internal Models):

DC signal:

\[ k \cdot u(t) \iff G_c(s) = \frac{k}{s + 0} \iff G_c(s) = 0 \mid_{s=j0} \]

Sinusoidal signal:

\[ k \cdot \cos(\omega t) \iff G_c(s) = \frac{ks}{s^2 + \omega^2} \iff G_c(s) = 0 \mid_{s=\pm j\omega} \]

It is clear that if the harmonic signal generators (internal models) are included in the controller \( G_c(s) \), \( G_c(s) \to \infty \) at the interested harmonic frequencies. Consequently, \( Y(s) \to R(s) \), i.e., zero-error tracking is achieved.
Internal Model Principle

- In the early 1970s, Francis, Wonham et al. laid the foundation of regulation theory with the Internal Model Principle which states that perfect asymptotic rejection/tracking of persistent inputs can only be attained by replicating the signal generator in a stable feedback loop.

- Wonham summarized the internal model principle: “Every good regulator must incorporate a model of the outside world”. 
Internal Model Principle based PID control:

Stability and dynamics

Internal model for DC signals
Control accuracy

Stability and dynamics
Internal Model Principle based periodic control:

Internal model for periodic signals (Resonant and repetitive control)
Control accuracy

Stability and dynamics

Introducing
Periodic Control for Power Electronic Converters
Questions?
Periodic signal generator (**Internal Models** of all harmonics):

\[
\hat{G}_{rc}(s) = \frac{e^{-sT_0}}{1-e^{-sT_0}} = \frac{1}{2} + \frac{1}{T_0 s} + \frac{1}{T_0} \sum_{n=1}^{\infty} \frac{2s}{s^2 + (n\omega_0)^2}
\]
Internal Model of Any Periodic Signal

Periodic signal generator (**Internal Models of all harmonics**):

\[ T_0 = 0.02 \, \text{s} \]
Development of Conventional Repetitive Control (CRC):

\[
G_{rc}(s) = \frac{k_{rc} Q(s) e^{-sT_0}}{1 - Q(s) e^{-sT_0}} e^{sT_c} \rightarrow k_{rc} e^{sT_c} \left[ -\frac{1}{2} + \frac{1}{T_0 s} + \frac{1}{T_0} \sum_{n=1}^{\infty} \frac{2s}{s^2 + (n\omega_0)^2} \right]
\]

- Control gain \( k_{rc}/T_0 \) for all frequencies: **identical convergence rate**
- Time lead \( T_c \) at all harmonics: **increase stability**
- \( Q(s) \) is usually a low pass filter: **increase stability**
Digital periodic signal generator \((\text{Internal Models of all harmonics})\):

\[
\hat{G}_{rc}(z) = \frac{Z^{-N}}{1 - Z^{-N}} = \frac{Z^{-T_0/T_s}}{1 - Z^{-T_0/T_s}}
\]
Conventional RC Scheme in the discrete-time domain:

\[ G_{rc}(z) = \frac{k_{rc}Q(z)z^{-N}}{1-Q(z)z^{-N}}G_f(z) \]

- \( \omega \to \pm i\omega_0, \ i = 0, 1, \ldots, N/2, \text{ or } (N-1)/2 \), \( G_{rc}(z) \to \infty \)
- **Identical gain** at all harmonics: \( k_{rc} \times 2/N \)
General “PID” System (digital RC + feedback control):

The feedback control system $H(z) = \frac{G_c(z)G_p(z)}{1 + G_c(z)G_p(z)}$ is stable

$|Q(z)(1 - k_r G_f(z) H(z))| < 1$
Achievable **Zero-Phase Compensation**:

Assuming \( H(z) = \frac{B(z)}{A(z)} = \frac{z^{-d}B^+(z)B^-(z)}{A(z)} \)

If \( G_f(z) = \frac{z^d A(z)B^-(z^{-1})}{B^+(z)b} \) with \( b \geq \max |B^-(e^{j\omega})|^2 \), and \( |Q(z)| \leq 1 \)

Then, \( G_f(z)H(z) = \frac{B^-(z)B^-(z^{-1})}{b} = \frac{|B^-(e^{j\omega})|^2}{b} \angle 0^\circ \leq 1 \)

Zero-Phase Compensation is achieved.

**Stability range** of the control gain:

\[
|Q(z)(1 - k_{rc}G_f(z)H(z))| < 1 \Rightarrow 1 - k_{rc} \left| \frac{B^-(z)}{b} \right|^2 < \frac{1}{|Q(z)|}
\]

0 < \( k_{rc} < 2 \)
Plug-in Digital CRC System

Linear Phase Compensation Design for the CRC system:

In practice, it is impossible to obtain an accurate transfer function of \( H(z) \),

\[
G_f(z)H(z) = \frac{B(z)B^{-1}(z^{-1})}{b}(1 + \Delta(z)) = |G_{fH}(e^{j\omega})| e^{j\theta_{fH}(\omega)} \quad \text{with} \quad |\Delta(z)| \leq \varepsilon
\]

\( \theta_{fH}(\omega) \neq 0 \)
Linear Phase Compensation Design for the CRC system:

To simplify the design, a linear phase-lead compensator $G_f(z)$ is introduced:

$$G_f(z) = z^p$$

$$G_{rc}(z) = k_{rc} \frac{Q(z)z^{-N+p}}{1 - Q(z)z^{-N}}$$

- Linear phase-lead compensator: **simplest but effective**
- At all harmonics, **identical lead steps**: not zero-phase compensation and reduced stability range of $k_{rc}$
Linear Phase Compensation – an example:

If we have a feedback control system \( H(z) = \frac{0.5z + 0.432}{z^2 - 0.487z + 0.429} \) with \( f_s = 10 \text{ kHz} \),

\[
2k\pi - \frac{\pi}{2} < \theta_H + p\omega \leq 2k\pi + \frac{\pi}{2} \quad \Rightarrow \quad 0 < k_{rc} < 1.1
\]
Internal Model for a specific harmonic of interest:

Any periodic signal can be decomposed into the sum of a set of harmonics (i.e., cosines and sines) and its DC component. The internal model of a periodic signal is equivalent to the sum of the internal models of its harmonics and DC component.

\[
\begin{align*}
\hat{G}_{h_1}(s) &= \mathcal{L}\{\cos(\omega_h t)\} = \frac{s}{s^2 + \omega_h^2} = \frac{1}{2} \left( \frac{1}{s + j\omega_h} + \frac{1}{s - j\omega_h} \right) \\
\hat{G}_{h_2}(s) &= \mathcal{L}\{\sin(\omega_h t)\} = \frac{\omega_h}{s^2 + \omega_h^2} = \frac{1}{2} \left( \frac{j}{s + j\omega_h} - \frac{j}{s - j\omega_h} \right)
\end{align*}
\]

Internal models of the selected harmonics approach to infinity at harmonic frequencies $\pm\omega_h$. Therefore, zero-error tracking of periodic signals can be achieved at frequencies of $\pm\omega_h$. 
Development of **Resonant Control (RSC):**

\[ G_h(s) = \mathcal{L}\{k_h \cos(\omega_h t + \theta_h)\} = k_h \frac{s \cos \theta_h - \omega_h \sin \theta_h}{s^2 + \omega_h^2} \]

- Control gain \( k_h \) for the harmonic: **convergence rate tuning**
- Phase-lead compensation \( \theta_h \): **system stability**
- No need for the low pass filter \( Q(s) \) as in the repetitive control
Parallel Resonant Control (MRSC) for multiple harmonics:

\[
G_M(s) = \sum_{h \in N_h} G_h(s) = \sum_{h \in N_h} \left( k_h \frac{s \cos \theta_h - \omega_h \sin \theta_h}{s^2 + \omega_h^2} \right)
\]

Digital Implementation:

\[
G_M(z) = \sum_{h \in N_h} k_h \left\{ \frac{1}{2} \left(1 - z^{-2}\right) \cos \theta_h \sin(\omega_h T_s) - \left(1 + 2z^{-1} + z^{-2}\right) \sin \theta_h \sin^2 \left(\frac{\omega_h T_s}{2}\right) \right\}
\]
Plug-in Digital MRSC System

Plug-in MRSC enabling selective harmonic cancellation:

Stability Conditions:

- Roots of $1 + G_c(z)G_p(z) = 0$ are inside the unit circle, i.e., $H(z)$ is stable
- Roots of $1 + G_M(z)H(z) = 0$ are inside the unit circle
Periodic Control of CVCF single-phase PWM inverters:

![Diagram of Inverter and Rectifier with State Feedback Controller](image-url)
### Periodic Control of CVCF single-phase PWM inverters:

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Nominal value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>DC-link voltage $v_{dc}$</td>
<td>250</td>
<td>V</td>
</tr>
<tr>
<td>Inductor filter $L_f$</td>
<td>3.3</td>
<td>mH</td>
</tr>
<tr>
<td>Capacitor filter $C_f$</td>
<td>100</td>
<td>µF</td>
</tr>
<tr>
<td>Resistive load $R$</td>
<td>60</td>
<td>Ω</td>
</tr>
<tr>
<td>Rectifier inductor $L_r$</td>
<td>3.3</td>
<td>mH</td>
</tr>
<tr>
<td>Rectifier capacitor $C_r$</td>
<td>1000</td>
<td>µF</td>
</tr>
<tr>
<td>Rectifier resistor $R_r$</td>
<td>60</td>
<td>Ω</td>
</tr>
<tr>
<td>Switching frequency</td>
<td>10</td>
<td>kHz</td>
</tr>
<tr>
<td>Sampling frequency</td>
<td>10</td>
<td>kHz</td>
</tr>
<tr>
<td>Reference voltage $v_c^*$</td>
<td>$155.6\sin(100\pi t)$</td>
<td>V</td>
</tr>
</tbody>
</table>
Periodic Control of CVCF single-phase PWM inverters:

\[
\begin{align*}
\dot{v}_c(k) &= -27.76 \frac{v_c(k)}{v_{dc}} + 4.15 \times 10^{-3} \frac{\dot{v}_c(k)}{v_{dc}} + 28.76 \frac{v_c^*(k)}{v_{dc}} \\
u(k) &= -\left[27.76 \frac{v_c(k)}{v_{dc}} + 4.15 \times 10^{-3} \frac{\dot{v}_c(k)}{v_{dc}}\right] + 28.76 \frac{v_c^*(k)}{v_{dc}}
\end{align*}
\]
Application Case – Results

State Feedback Control of the CVCF single-phase PWM inverter: with a fundamental-frequency RSC

![Graph showing voltage and current waveforms with harmonic analysis and THD calculation.](image)
Application Case – Results

State Feedback Control of the CVCF single-phase PWM inverter: with the repetitive control (i.e., RC)
Application Case – Results

State Feedback Control of the CVCF single-phase PWM inverter:
with multiple resonant controllers (i.e., MRSC)

![Graph showing voltage and current responses with harmonic analysis]
DFT-based Repetitive Control

DFT-based Band-Pass Filter of selected harmonics:

Discrete Fourier Transform

\[ F_{dh}(z) = \frac{2}{N} \sum_{i=1}^{N} \cos\left(\frac{2\pi}{N} h(i + N_a)\right)z^{-i} \approx Q_{dh}(z)z^{N_a} \]

\( N = 100, \ N_a = 0 \)
DFT-based Band-Pass Filter of selected harmonics: Discrete Fourier Transform

\[ F_{dh}(z) = \frac{2}{N} \sum_{i=1}^{N} \cos \left( \frac{2\pi}{N} h(i + N_a) \right) z^{-i} \approx Q_{dh}(z) z^{N_a} \]

\[ F_{DFT}(z) = \sum_{h \in N_h} F_{dh}(z) = \frac{2}{N} \sum_{i=1}^{N} \left[ \sum_{h \in N_h} \cos \left( \frac{2\pi}{N} h(i + N_a) \right) \right] z^{-i} \]

\[ F_{DFT}(z) = \left( \sum_{i=1}^{N} b_h(i) z^{-i} \right) z^{N_a} = Q_D(z) z^{N_a} \]
DFT-based Repetitive Control

**DFT-based Band-Pass Filter** of selected harmonics:

Discrete Fourier Transform

\[
F_{\text{DFT}}(z) = \left( \sum_{i=1}^{N} b_h(i) z^{-i} \right) z^{N_0} = Q_D(z) z^{N_0}
\]

A **Comb Filter** is developed.
DFT-based Internal Model of selected harmonics: Discrete Fourier Transform

\[ F_{\text{DFT}}(z) = \left( \sum_{i=1}^{N} b_h(i) z^{-i} \right) z^{N_a} = Q_D(z) z^{N_a} \]

\[ \hat{G}_{\text{DFT}}(z) = \frac{F_{\text{DFT}}(z)}{1 - F_{\text{DFT}}(z)} \]

For example, if \( N = 100 \), \( N_a = 0 \), and \( h = 0, 1, 2, \ldots, 49 \) (all pass), then

\[ F_{\text{DFT}}(z) = \frac{2}{N} \sum_{i=1}^{100} \left( \sum_{h=0}^{49} \cos\left( h \frac{2\pi i}{N} \right) \right) z^{-i} = z^{-100} \]

\[ \hat{G}_{\text{DFT}}(z) = \frac{z^{-100}}{1 - z^{-100}} \]
DFT-based Repetitive Control scheme:

\[ G_{DFT}(z) = \frac{u_{rc}(z)}{e(z)} = k_F \frac{F_{DFT}(z)}{1 - F_{DFT}(z)z^{-N_a}} = k_F \frac{Q_D(z)}{1 - Q_D(z)} z^{N_a} \]

- Control gain \(2k_F/N\) for all frequencies: **identical convergence rate**
- Phase lead step \(N_a\) at all harmonics: **increase stability**
Plug-in DFT-based Repetitive Control

Plug-in DFT-based RC System compatible periodic control:

Stability Conditions:

- Roots of $1 + G_c(z)G_p(z) = 0$ are inside the unit circle, i.e., $H(z)$ is stable
- Roots of $1 + G_{DFT}(z)H(z) = 0$ are inside the unit circle

Design of the plug-in DFT-based RC system is similar to other plug-in periodic control systems.
Modified DFT-based Repetitive Control scheme:

Discrete Fourier Transform

\[
F'_{dh}(z) = \frac{2}{N} \sum_{i=1}^{N} a_h \cos \left( \frac{2\pi}{N} h(i + N_a) \right) z^{-i} = Q'_{dh}(z) z^{N_a}
\]

\[
F'_{DFT}(z) = \sum_{h \in N_h} F'_{dh}(z) = \sum_{h \in N_h} \left\{ \frac{2}{N} \sum_{i=1}^{N} a_h \cos \left( \frac{2\pi}{N} h(i + N_a) \right) z^{-i} \right\} = Q'_D(z) z^{N_a}
\]

\[
G'_{DFT}(z) = \frac{u_{rc}(z)}{e(z)} = k_F \frac{F'_{DFT}(z)}{1 - F'_{DFT}(z) z^{-N_a}} = k_F \frac{Q'_D(z)}{1 - Q'_D(z)} z^{N_a}
\]

- Control gain \(2k_F a_h/N\) for the \(h\)-order harmonic: tune for proper convergence rate
- Phase lead step \(N_a\) at all harmonics is still identical
MRSC scheme \( \approx \) DFT-based RC scheme:

Since MRSC \( G_M(z) \) offers more degrees of freedom in adopting both independent gain and independent phase-lead compensation for each harmonic, when compared with the modified DFT-based RC \( G'_{DFT}(z) \).

That’s to say, the modified DFT-based RC is actually a special case of the MRSC. Hence, \( G_M(z) \) can be roughly approximated by \( G'_{DFT}(z) \).

\[
G_M(z) = \sum_{h \in N_h} G_h(z) \approx k_F \frac{Q'_D(z)}{1 - Q'_D(z)} z^{N_a}
\]

\[
k_h \approx k_F \cdot \frac{2a_h}{N}
\]
RSC Scheme ≡ I scheme in the synchronous rotating frame:

Zero-error tracking can be achieved using PI controllers in the stationary reference frame, and also using PR controllers in the synchronous rotating frame.

$$G_{dq}^+(s) = G_{dq}^-(s) = G_{dq}(s) = \begin{pmatrix} k_i/s & 0 \\ 0 & k_i/s \end{pmatrix}$$

$$G_{a\beta}(s) = G_{a\beta}^+(s) + G_{a\beta}^-(s) = \begin{pmatrix} \frac{k_is}{s^2 + \omega^2} & \frac{k_i\omega}{s^2 + \omega^2} & \frac{k_i\omega}{s^2 + \omega^2} & \frac{2k_is}{s^2 + \omega^2} \\ \frac{k_i\omega}{s^2 + \omega^2} & -\frac{k_i\omega}{s^2 + \omega^2} & \frac{k_is}{s^2 + \omega^2} & 0 \\ \frac{2k_is}{s^2 + \omega^2} & 0 & \frac{2k_is}{s^2 + \omega^2} & \frac{2k_is}{s^2 + \omega^2} \end{pmatrix}$$
RSC Scheme $\equiv$ I scheme in the synchronous rotating frame:

Zero-error tracking can be achieved using PI controllers in the stationary reference frame, and also using PR controllers in the synchronous rotating frame.

A PR controller is equivalent to the combination of two PI controllers.
Comparison

Conventional Repetitive Control

W/O consideration of the harmonic distribution in power converters

- **Accurate**: compensate any known periodic signal
- **Recursive**: compact form, light computation, easy-implementation
- **Slow**: limited gain. It’s impossible to optimize its transient response by tuning gains independently at selected harmonic frequencies.
Multiple Resonant Control

Considering the harmonic distribution, multiple RSC components with independent gain $k_h$ and phase lead compensation $\theta_h$ at each harmonic frequency

- **Paralleled connection**: multiple RSC components can yield high control accuracy. However, too many RSC components will yield heavy parallel computation burden and tuning difficulty in implementation.

- **Independent gain** (and much larger) $k_h$ and **phase lead compensation** $\theta_h$ enable MRSC to optimize its transient response and stability.
Comparison

DFT-based Repetitive Control

Considering the harmonic distribution in power converters, multiple selective harmonics with identical or independent gains and identical phase lead step $N_a$

- **Compatible phase delay compensation**: equivalent to linear phase-lead compensation RC scheme.
- **Dynamic optimization**: modified DFT-based RC allows users to optimize its dynamics by tuning coefficients (i.e. gains) at selected harmonics.
- **Flexible harmonic compensation**: a large amount of parallel computation for implementation, which is proportional to the fundamental period $N$. It may be suitable for high performance fixed-point DSP implementation.
Fundamentals in Periodic Control:

- **CRC, MRSC, and DFT-based RC** are the fundamental periodic control schemes.
- **Compatible stability criteria** are achieved for the three plug-in fundamental periodic control systems.
- **General “PID” control scheme** is formed by combing the feedback control and fundamental periodic control.
- **Optimal periodic control** is needed to achieve fast dynamics, high accuracy, good compatibility, and easy-for-implementation.
Fundamentals in Periodic Control:

- **CRC, MRSC, and DFT-based RC** are the fundamental periodic control schemes.
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Introducing

**Advanced Periodic Control**
Questions?
10 Minutes
Harmonics are **Unevenly Distributed** in Power Converters:

*n*-pulse converters produce dominant $nk\pm 1$ ($k = 0, 1, ...$) order harmonics due to

*n*-pulse commutation
A comparison of *RC* and *MRSC* schemes:

**Repetitive Control**
- Recursive form
- Internal models of all harmonics
- Identical gain for all harmonics
- Accurate but relatively slow dynamic response

**Multiple Resonant Control**
- Parallel structure
- Only internal models of the selected harmonics
- Can optimize gains for the selected harmonics
- Fast but heavy parallel computation burden

How to **optimize periodic controllers** for selective harmonic mitigation for high accuracy, fast dynamics, cost-effective and easy implementation?
Complex Internal Model of selected harmonics:
Complex Internal Model of selected harmonics:

\[
\hat{G}_{\pm m}(s) = \frac{u(s)}{e(s)} = \frac{e^{-2\pi \left[\frac{s}{(n\omega_0)} \mp j(m/n)\right]}}{1 - e^{-2\pi \left[\frac{s}{(n\omega_0)} \pm j(m/n)\right]}} \\
= -\frac{1}{2} + \frac{n}{T_0} \cdot \frac{1}{s \mp jm\omega_0} + \frac{2n}{T_0} \sum_{k=1}^{\infty} \frac{s \mp jm\omega_0}{(s \mp jm\omega_0)^2 + n^2 k^2 \omega_0^2}
\]
Complex Internal Model of selected harmonics:

\[ T_0 = 0.02 \, s \quad 6k+1 \]
Take the advantages of **RC** and **MRSC** schemes:

\[
G_{psrc}(s) = \sum_{m=0}^{n-1} k_{pm} \hat{G}_m(s) = \sum_{m=0}^{n-1} \left\{ k_{pm} \frac{e^{-2\pi [s/(n\omega_0) - j(m/n)]}}{1 - e^{-2\pi [s/(n\omega_0) - j(m/n)]}} \right\}
\]

If \( k_{pm} = k_{rc}/n \), then

\[
G_{psrc}(s) = \sum_{m=0}^{n-1} \left[ \frac{k_{rc}}{n} \hat{G}_m(s) \right]
\]

\[
= \sum_{m=0}^{n-1} \left\{ \frac{k_{rc}}{n} \left[ -\frac{1}{2} + \frac{n}{T_0} \cdot \frac{1}{s - jm\omega_0} + \frac{2n}{T_0} \sum_{k=1}^{\infty} \frac{s - jm\omega_0}{(s - jm\omega_0)^2 + n^2k^2\omega_0^2} \right] \right\}
\]

\[
= k_{rc} \left[ -\frac{1}{2T_0s} + \frac{1}{T_0s^2} + \frac{2}{T_0} \sum_{k=1}^{\infty} \frac{s}{s^2 + (n\omega_0)^2} \right] = k_{rc} \frac{e^{-sT_0}}{1 - e^{-sT_0}} = G_{rc}(s)
\]
Parallel Structure Repetitive Control

Take the advantages of **RC and MRSC** schemes:

\[
G_{\text{psrc}}(s) = \sum_{m=0}^{n-1} k_{pm} \hat{G}_m(s) = \sum_{m=0}^{n-1} k_{pm} \frac{e^{-2\pi s/(n\omega_0) - j(m/n)}}{1 - e^{-2\pi s/(n\omega_0) - j(m/n)}}
\]

In practice, a low-pass or band-pass filter \(Q_{m}(s)\) and a phase-lead compensator \(G_{f}(s)\) are adopted,

\[
G_{\text{psrc}}(s) = \sum_{m=0}^{n-1} k_{pm} \hat{G}'_m(s) G_{f}(s)
\]

\[
= \sum_{m=0}^{n-1} \left\{ k_{pm} \frac{e^{-2\pi s/(n\omega_0) - j(m/n)}}{1 - e^{-2\pi s/(n\omega_0) - j(m/n)}} \frac{Q_{m}(s)}{Q_{m}(s)} G_{f}(s) \right\}
\]

Further, let \(k_{pm} = k_{rc}/n\) and \(Q_{m}(s) = Q(s)\),

\[
G_{\text{psrc}}(s) = \frac{k_{rc}}{n} \sum_{m=0}^{n-1} \left\{ \frac{e^{-2\pi s/(n\omega_0) - j(m/n)}}{1 - e^{-2\pi s/(n\omega_0) - j(m/n)}} \frac{Q(s)}{Q(s)} G_{f}(s) \right\} = k_{rc} \frac{e^{-sT_0} Q^n(s)}{1 - e^{-sT_0} Q^n(s)} G_{f}(s)
\]
Parallel Structure Repetitive Control

Take the advantages of **RC and MRSC** schemes:

\[ G_{\text{psrc}}(s) = \frac{k_{rc}}{n} \sum_{m=0}^{n-1} \left\{ \frac{e^{-2\pi s / (n\omega_0) - j(m/n)}}{1 - e^{-2\pi s / (n\omega_0) - j(m/n)}} Q(s) G_f(s) \right\} = k_{rc} \frac{e^{-sT_0} Q^n(s)}{1 - e^{-sT_0} Q^n(s)} G_f(s) \]

The parallel structure repetitive control \( G_{\text{psrc}}(s) \) is **equivalent** to the conventional repetitive control \( G_{rc}(s) \) when \( k_{pm} = k_{rc}/n \) and \( Q_m(s) = Q(s) \).
Take the advantages of **RC and MRSC** schemes:

\[
G_{psrc}(z) = \sum_{m=0}^{n-1} \left[ k_{pm} \hat{G}_m(z) G_f(z) \right] = \sum_{m=0}^{n-1} \left\{ k_{pm} \frac{e^{j(2\pi m/n)} z^{-N/n} Q_m(z)}{1 - e^{j(2\pi m/n)} z^{-N/n} Q_m(z)} G_f(z) \right\}
\]
Plug-in Digital PSRC System

Take the advantages of **RC and MRSC** schemes:

**Stability Conditions:**

- Roots of $1 + G_c(z)G_p(z) = 0$ are inside the unit circle, i.e., $H(z)$ is stable
- Roots of $1 + G_{psrc}(z)H(z) = 0$ are inside the unit circle

\[
0 < \sum_{m=0}^{n-1} k_{pm} < 2
\]
Real Internal Model of selected harmonics:

\[
\hat{G}_{sm}(s) = \frac{1}{2}(\hat{G}_m(s) + \hat{G}_m(-s)) = \frac{1}{2}\left\{ \frac{e^{-2\pi[s/(n\omega_0)-j(m/n)]}}{1-e^{-2\pi[s/(n\omega_0)-j(m/n)]}} + \frac{e^{-2\pi[s/(n\omega_0)+j(m/n)]}}{1-e^{-2\pi[s/(n\omega_0)+j(m/n)]}} \right\}
\]

\[
\cos(2\pi m / n)e^{2\pi s/(n\omega_0)} - 1
\]

\[
e^{4\pi s/(n\omega_0)} - 2\cos(2\pi m / n)e^{2\pi s/(n\omega_0)} + 1
\]
Real Internal Model of selected harmonics:

Internal models for $6k \pm 1$ order harmonics

Internal models for $4k \pm 1$ order harmonics
Real Internal Model of selected harmonics:

$T_0 = 0.02 \, s \quad 6k \pm 1$
Using real internal models for Selective Harmonic Control:

\[
G_{sm}(s) = \frac{k_m}{2} \left[ \frac{e^{-2\pi s/(n\omega_b-j(m/n))} Q(s)}{1-e^{-2\pi s/(n\omega_b-j(m/n))} Q(s)} + \frac{e^{-2\pi s/(n\omega_b+j(m/n))} Q(s)}{1-e^{-2\pi s/(n\omega_b+j(m/n))} Q(s)} \right] G_f(s)
\]

\[
= k_m \frac{\cos(2\pi m/n)e^{2\pi s/(n\omega_b)} Q(s) - Q^2(s)}{e^{4\pi s/(n\omega_b)} - 2\cos(2\pi m/n)e^{2\pi s/(n\omega_b)} Q(s) + Q^2(s)} G_f(s)
\]
Using real internal models for Selective Harmonic Control:

\[
G_{sm}(z) = k_m \frac{\cos(2\pi m / n)Q(z)z^{N/n} - Q^2(z)}{z^{2N/n} - 2\cos(2\pi m / n)Q(z)z^{N/n} + Q^2(z)} G_f(z)
\]
Enabling **Fast Dynamics** – Selective harmonic control scheme:

**Stability Conditions:**

- Roots of $1 + G_c(z)G_p(z) = 0$ are inside the unit circle, i.e., $H(z)$ is stable.
- Roots of $1 + G_{sm}(z)H(z) = 0$ are inside the unit circle, i.e.,

\[ |Q^2(z)(1-k_m G_f(z)H(z))| < 1 \]

\[ 0 < k_m < 2 \]
Enabling **Fast Dynamics** – Selective harmonic control scheme:

In practice, a linear phase-lead compensator $G_f(z) = z^c$ is adopted.
### Comparison

**A comparison of SHC and RC schemes:**

<table>
<thead>
<tr>
<th></th>
<th>Selective Harmonic Control</th>
<th>Repetitive Control</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stability range</td>
<td>$0 &lt; k_m &lt; 2$</td>
<td>$0 &lt; k_{rc} &lt; 2$</td>
</tr>
<tr>
<td>Equivalent gain</td>
<td>$nk_m \frac{1}{2} \frac{1}{T_0}$</td>
<td>$k_{rc} \frac{1}{T_0}$</td>
</tr>
</tbody>
</table>

If $k_m = k_{rc}$

At the selected $nk \pm m$ order harmonics,

Convergence rate of SHC is $n/2$ times faster than that of RC.
Odd Order Harmonic RC – an SHC for single-phase converters:

Internal models for $4k\pm1$ order harmonics

At $4k\pm1$ order harmonics, the convergence rate of SHC is 2 times faster than that of RC. This scheme is especially suitable for single-phase (4-pulse) converters.
6k±1 Order Harmonic Repetitive Control

Internal models for 6k±1 order harmonics

\[ G_{rc}(z) = -k_{rc} \frac{z^{-N/6+c}Q(z) / 2 - z^{-N/3+c}Q^2(z)}{1 - z^{-N/6}Q(z) + z^{-N/3}Q^2(z)} \]

At 6k±1 order harmonics, the convergence rate of SHC is 3 times faster than that of RC. This scheme is especially suitable for three-phase (6-pulse) converters.
Optimized Gain for each selective harmonic control module:

\[
G_{sm}(z) = k_m \frac{\cos(2\pi m / n)Q(z)z^{N/n} - Q^2(z)}{z^{2N/n} - 2\cos(2\pi m / n)Q(z)z^{N/n} + Q^2(z)} G_f(z)
\]
Optimized Gain for each selective harmonic control module:

\[
G_{OHC}(z) = \sum_{m \in N_m} G_{sm}(z) = \sum_{m \in N_m} \left[ k_m \frac{\cos(2\pi m / n)Q(z)z^{N/n} - Q^2(z)}{z^{2N/n} - 2\cos(2\pi m / n)Q(z)z^{N/n} + Q^2(z)} G_f(z) \right]
\]
**Optimally Weighted Gain** leads to fast dynamics:

Stability Conditions:

- Roots of $1 + G_c(z)G_p(z) = 0$ are inside the unit circle, i.e., $H(z)$ is stable
- $0 < \sum_{m \in N_m} k_m < 2$ and $k_m \geq 0$

Control gain for each selective harmonic control module $G_{sm}(z)$ can be **optimally weighted** (e.g., according to the harmonic distribution) $\Rightarrow$ **fast dynamics**
**Dual-Module RC Scheme** – an OHC for single-phase converters:

Internal models for $4k\pm1$ and $4k\pm2$ order harmonics

$$G_{orc}(z) = -k_{or} \frac{z^{-N/2+c}Q_o(z)}{1+z^{-N/2}Q_o(z)}$$

$$G_{erc}(z) = k_{er} \frac{z^{-N/2+c}Q_e(z)}{1-z^{-N/2}Q_e(z)}$$

$$G_{DMRC}(z) = \left[ -k_{or} \frac{z^{-N/2}Q_o(z)}{1+z^{-N/2}Q_o(z)} + k_{er} \frac{z^{-N/2}Q_e(z)}{1-z^{-N/2}Q_e(z)} \right] z^c$$

- Odd-order Harmonics
- Even-order Harmonics
Dual-Module RC Scheme – an OHC for single-phase converters:

\[ G_{DMRC}(z) = -k_{or} \frac{z^{-N/2+c}Q_o(z)}{1 + z^{-N/2}Q_o(z)} + k_{er} \frac{z^{-N/2+c}Q_e(z)}{1 - z^{-N/2}Q_e(z)} \]

Convergence rate of Dual-Module RC is up to 2 times faster than that of RC, a universal PC scheme for single-phase converters.
Application Case

Advanced Periodic Control of CVCF single-phase PWM inverters:

[Diagram of inverter and rectifier systems with state feedback controller and reference voltage]
**Application Case**

## Advanced Periodic Control of CVCF single-phase PWM inverters:

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Nominal value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>DC-link voltage $v_{dc}$</td>
<td>80</td>
<td>V</td>
</tr>
<tr>
<td>Inductor filter $L_f$</td>
<td>20</td>
<td>mH</td>
</tr>
<tr>
<td>Capacitor filter $C_f$</td>
<td>45</td>
<td>µF</td>
</tr>
<tr>
<td>Resistive load $R$</td>
<td>15</td>
<td>Ω</td>
</tr>
<tr>
<td>Rectifier inductor $L_r$</td>
<td>1</td>
<td>mH</td>
</tr>
<tr>
<td>Rectifier capacitor $C_r$</td>
<td>500</td>
<td>µF</td>
</tr>
<tr>
<td>Rectifier resistor $R_r$</td>
<td>22</td>
<td>Ω</td>
</tr>
<tr>
<td>Switching frequency</td>
<td>10</td>
<td>kHz</td>
</tr>
<tr>
<td>Sampling frequency</td>
<td>10</td>
<td>kHz</td>
</tr>
<tr>
<td>Reference voltage $v_c^*$</td>
<td>50sin(100πt)</td>
<td>V</td>
</tr>
</tbody>
</table>
Advanced Periodic Control of CVCF single-phase PWM inverters:

\[
\begin{align*}
 u(k) &= - \left[ 90 \times \frac{v_c(k)}{v_{dc}} + 8.4 \times 10^{-3} \times \frac{\dot{v}_c(k)}{v_{dc}} \right] + 90 \times \frac{v_c^*(k)}{v_{dc}}
\end{align*}
\]
Application Case – Results

**State Feedback Control** of the CVCF single-phase PWM inverter: without any advanced periodic control
**Application Case – Results**

**State Feedback Control** of the CVCF single-phase PWM inverter: without any advanced periodic control

\[
h_r(j) = \frac{\sum_{i=1}^{200} M_i}{\sum_{i=1}^{199} M_i} \times 100\%
\]

\[
Q(z) = 0.25z + 0.5 + 0.25z^{-1}
\]

- **Harmonic ratio (%)**
- **Frequency (kHz)**: 0 to 5
- **Magnitude (dB)**: -50 to 0

1800 Hz

1820 Hz

- **95%**
Application Case – Results

**State Feedback Control** of the CVCF single-phase PWM inverter: with various **advanced periodic control**

- **RC**, $k_{rc} = 1.2$
- **ORC**, $k_{or} = 1.2$
- **DMRC**, $k_{or} = 0.4$, $k_{er} = 0.8$
- **DMRC**, $k_{or} = 0.8$, $k_{er} = 0.4$
State Feedback Control of the CVCF single-phase PWM inverter:

with various advanced periodic control

RC; $k_{rc} = 1.2$

ORC; $k_{or} = 1.2$

DMRC; $k_{or} = 0.4, k_{er} = 0.8$

DMRC; $k_{or} = 0.8, k_{er} = 0.4$

Time [50 ms/div]
**State Feedback Control** of the CVCF single-phase PWM inverter: with various **advanced periodic control**

<table>
<thead>
<tr>
<th>Periodic Control</th>
<th>THD, %</th>
<th>Speed, s</th>
<th>Even harmonics?</th>
</tr>
</thead>
<tbody>
<tr>
<td>Repetitive control, $k_{rc} = 1.2$</td>
<td>0.8</td>
<td>0.2</td>
<td>YES</td>
</tr>
<tr>
<td>Odd-harmonic RC, $k_{or} = 1.2$</td>
<td>1.2</td>
<td>0.1</td>
<td>NO</td>
</tr>
<tr>
<td>Dual-module RC, $k_{or} = 0.4$, $k_{er} = 0.8$</td>
<td>0.8</td>
<td>0.32</td>
<td>YES</td>
</tr>
<tr>
<td>Dual-module RC, $k_{or} = 0.8$, $k_{er} = 0.4$</td>
<td>0.5</td>
<td>0.16</td>
<td>YES</td>
</tr>
</tbody>
</table>
Summary

Selective Harmonic Control:

Stability criterion of SHC is compatible to that of RC. When selected harmonics dominate the tracking errors,

- SHC can be used to achieve much faster convergence rate than RC
- SHC occupies less computation resources than RC
- Tracking accuracy of SHC is a little less than that of RC
- Recursive form for easy-implementation
Summary

Optimal Harmonic Control:

Take the **advantages** of the RC and MRSC schemes, and it allows optimizing the control gains,

- OHC can achieve **high control accuracy** due to the removal of selected clusters of harmonics (up to all)
- OHC offers **fast dynamics** due to parallel combination of optimally weighted SHC modules
- Cost-effective and **easy real-time implementation** due to the universal recursive SHC modules
- Design is **compatible** with other periodic control schemes
Summary

Optimal Harmonic Control:

Take the **advantages** of the RC and MRSC schemes, and it allows optimizing the control gains,

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- OHC offers **fast dynamics** due to parallel combination of optimally weighted SHC modules
- Cost-effective and **easy real-time implementation** due to the universal recursive SHC modules
- Design is **compatible** with other periodic control schemes

Introducing

**Frequency Adaptive Periodic Control**
Periodic Control for grid-connected power converters:

- Implemented in low-cost digital control units;
  Control in various reference frames ($abc$, $dq$, and $\alpha\beta$)
- Currents should synchronize with the grid voltages;
- Grid frequency is **not constant**.

How will the controllers behave? What are the solutions?
**Frequency Dependency**

**Periodic Control** for grid-connected power converters:

- Implemented in low-cost digital control units;
- Control in **various reference frames** ($abc$, $dq$, and $αβ$);
- Currents should **synchronize** with the grid voltages;
- Grid frequency is **not constant**.

How will the controllers behave? What are the solutions?

---

![Graph showing frequency dependency over time](image-url)
Frequency Dependency

Performance of the periodic control is **Frequency-Dependent**:

- Implemented in low-cost digital controllers

\[ G_h(z) = \frac{k_i^h(z^{-1} - z^{-2})}{1 + \left(h^2 \frac{\omega_0}{2T_s^2} - 2\right)z^{-1} + z^{-2}} \]

\[ G_{RC}(z) = \frac{k_{rc}z^{-N}}{1 - z^{-N}} \]

- \( \omega_0 \) treated as a constant
- Fixed sampling frequency \( f_s \) (also \( T_s \)) for simplicity
- Grid frequency is time-varying →
  - i.e., \( \omega_{\text{PLL}} \) is not strictly constant
  - \( N = \frac{f_s}{f_0} = \frac{2\pi f_s}{\omega_0} \) will be a fractional
Performance of the periodic control is **Frequency-Dependent**:

Actual grid frequency can be expressed as

\[ \hat{\omega}_0 = \omega_0 + \Delta \omega = \omega_0 + \Delta \omega_g + \Delta \omega_{pll} \]

- **Frequency estimator errors**
- **Grid frequency changes**
Frequency Adaptability

Performance of the periodic control is **Frequency-Dependent**:

Actual grid frequency can be expressed as

\[
\hat{\omega}_0 = \omega_0 + \Delta \omega = \omega_0 + \Delta \omega_g + \Delta \omega_{pll}
\]

**Frequency sensitivity** of RSC and RC schemes can be obtained

\[
|G_h(jh\hat{\omega}_0)| = \frac{jk_hh\hat{\omega}_0}{-\left(h\hat{\omega}_0\right)^2 + \left(h\omega_0\right)^2} = \frac{k_h}{h\omega_0} \left| \frac{\delta + 1}{\delta^2 + \delta} \right|
\]

\[
|G_{RC}(jh\hat{\omega}_0)| = \frac{k_{rc}}{1 - Q(jh\hat{\omega}_0)e^{-2\pi(jh\hat{\omega}_0)/\omega_0}} = \frac{k_{rc}}{\sqrt{2 - 2\cos(2\pi h\delta)}}|Q(jh\hat{\omega}_0)|^{-1}
\]
Performance of the periodic control is **Frequency-Dependent**:

![Graph showing frequency adaptability](image)

It calls for

**Frequency Adaptive Periodic Control**
Frequency Adaptive Resonant Control

Directly **Feeding the Frequency** to the resonant control:

Frequency adaptive resonant control

\[
G_{ah}(s) = \frac{u_{rsc}(s)}{e(s)} = k_h \frac{s}{s^2 + \hat{\omega}_h^2}
\]

Considering phase compensation, the **frequency adaptive multiple resonant control** is obtained

\[
G_{aM}(s) = \sum_{h \in N_h} G_{ah}(s) = \sum_{h \in N_h} \left( k_h \frac{s \cos \theta_h - \hat{\omega}_h \sin \theta_h}{s^2 + \hat{\omega}_h^2} \right)
\]
Frequency Adaptive Repetitive Control

Impossible to implement $z^{-N}$ if $N$ is fractional:

Frequency adaptive repetitive control

$$G_{arc}(z) = k_{rc} \frac{Q(z)z^{-N}}{1 - Q(z)z^{-N}} G_f(z)$$

$$N = T_0 / T_s$$

Solution 1 - Variable Sampling Rate (VSR)

Ensuring $N$ is always a constant integer if frequency changes. VSR approach enables RC to compensate harmonics due to frequency variations.

- **Increased** the real-time implementation complexity, such as online controller redesign.
- **Cannot** deal with multiple signals with coprime frequencies simultaneously (It is impossible to ensure all $N_i$ to be integers).
Impossible to implement $Z^{-N}$ if $N$ is fractional:

Frequency adaptive repetitive control

$$G_{arc}(z) = k_{rc} \frac{Q(z)z^{-N}}{1 - Q(z)z^{-N}} G_f(z)$$

$N = T_0/T_s$

Solution 2 - Fractional Delay (FD) at fixed sampling rate

Fixed sampling rate significantly simplifies the design of the frequency adaptive RC scheme. Fractional delay (FD) filters can approximate the real delay.

- **Simple** in real-time implementation – minor software modifications.
- **Tolerate** large frequency variations (good portability).
- **Can** deal with multiple signals with coprime frequencies simultaneously
  (It is possible to approximate all $N_i$ at the same time).
Lagrange Interpolation for fractional delay filter:

\[ z^{-N} = z^{-N_i - F} = z^{-N_i} \cdot z^{-F} \]

- **Integer part**: Easy to implement
- **Fractional part**: Polynomial approximation

\[ z^{-F} \approx \sum_{k=0}^{n} A_k z^{-k} \quad \text{with} \quad A_k = \prod_{i=0}^{n} \frac{F - i}{k - i} \]

\[ z^{-N} \approx z^{-N_i} \cdot \sum_{k=0}^{n} A_k z^{-k} \]

- **Integer part**: Easy to implement
- **Approximated part**: Easy to implement
Lagrange Interpolation fractional delay filter:

FD filter with $n = 3$ gives an **excellent approximation** of $z^{-F}$ within bandwidth of 75% the Nyquist frequency; while 50% the Nyquist frequency, if $n = 1$. 
**Implement** frequency adaptive RC using the fractional delay:

\[
G_{\text{arc}}(z) = k_{\text{rc}} \frac{Q(z) \left( z^{-N_i} \cdot \sum_{k=0}^{n} A_k z^{-k} \right)}{1 - Q(z) \left( z^{-N_i} \cdot \sum_{k=0}^{n} A_k z^{-k} \right)} G_f(z)
\]

Frequency detector (e.g., PLL)

\[
f = \frac{f_s}{f} = N_i + F
\]

\[
A_k = \prod_{i=0}^{n} \frac{F - i}{k - i}
\]
Plug-in Digital FA-RC System

Frequency Adaptive RC (FA-RC) system:

Stability Conditions:

- Roots of $1 + G_c(z)G_p(z) = 0$ are inside the unit circle, i.e., $H(z)$ is stable
- Roots of $1 + G_{arc}(z)H(z) = 0$ are inside the unit circle

\[
\left| 1 - k_{rc} G_f(z) H(z) \right| < \frac{1}{Q(z) \left| \sum_{k=0}^{n} A_k z^{-k} \right|} \quad \Rightarrow \quad 0 < k_{rc} < 2
\]
Implement FA-PSRC scheme using the fractional delay:

\[
G_{apsrc}(z) = \sum_{m=0}^{n-1} k_{pm} \left\{ \frac{e^{j(2\pi m/n)} z^{-N_i} \cdot \sum_{k=0}^{L} A_k z^{-k} Q_m(z)}{1 - e^{j(2\pi m/n)} z^{-N_i} \cdot \sum_{k=0}^{L} A_k z^{-k} Q_m(z)} \right\} G_f(z)
\]
Implement FA-SHC scheme using the fractional delay:

\[
G_{asm}(z) = k_m \frac{\cos(2\pi m / n)Q(z)z^{-N_i} \cdot \sum_{k=0}^{L} A_k z^{-k} - Q^2(z) \left( z^{-N_i} \cdot \sum_{k=0}^{L} A_k z^{-k} \right)^2}{1 - 2\cos(2\pi m / n)Q(z)z^{-N_i} \cdot \sum_{k=0}^{L} A_k z^{-k} + Q^2(z) \left( z^{-N_i} \cdot \sum_{k=0}^{L} A_k z^{-k} \right)^2} \cdot G_f(z)
\]
Virtual Variable Sampling Rate Unit Delay:

\[
G_{\text{DFT}}(z) = k_F \frac{F_{\text{DFT}}(z)}{1 - F_{\text{DFT}}(z) z^{-N_a}}
\]
Virtual Variable Sampling Rate Unit Delay:

\[ N_F = \frac{f_s}{f} = N + F = N(1 + F_N) = \frac{f_s}{f_0}(1 + F_N) \]

\[ Z_v^{-1} = Z^{-(1 + F_N)} \]

With the linear Lagrange interpolation method

\[ Z_v^{-1} = Z^{-(1 + F_N)} \approx \begin{cases} F_N |Z|^0 + (1 - |F_N|)Z^{-1} & -1 < F_N < 0 \\ (1 - |F_N|)Z^{-1} + |F_N|Z^{-2} & 0 \leq F_N < 1 \end{cases} \]

\[ F_{aDFT}(z) = \left( \sum_{i=1}^{N} b_h(i) z_v^{-i} \right) z_v^{N_a} \]
Frequency Adaptive DFT RC using virtual unit delay $Z_v^{-1}$:

$$G_{aDFT}(z) = \frac{k_F F_{aDFT}(z)}{1 - F_{aDFT}(z) Z_v^{-N_a}}$$

\[ aDFT \]

\[ aDFT \]

\[ aDFT \]

\[ aDFT \]

Virtual Unit Delay $Z_v^{-1}$
Fractional-Order Linear Phase Compensation:

\[ \Phi = c \times \frac{f}{f_s} \times 360^\circ \]
Fractional-Order Linear Phase Compensation:

If $\phi$ is not an integer (i.e., fractional phase compensation is required) or due to frequency variations, the phase lead compensation $z^c$ is **not accurate**, $c$ can be a **fractional number**. This can not be implemented in a fixed sampling rate system.

$$\Phi = c \times \frac{f}{f_s} \times 360^\circ$$
Fractional-Order Linear Phase Compensation:

If $\phi$ is not an integer (i.e., fractional phase compensation is required) or due to frequency variations, the phase lead compensation $z^c$ is not accurate, $c$ can be a fractional number. This can not be implemented in a fixed sampling rate system.

Alternatively,

$$G_f(z) = z^c = z^{n_i+F} \approx (1-F)z^{n_i} + Fz^{n_i+1}$$

Then, a fractional $c$ yields flexible phase lead compensation $(\theta_H + c\omega)$ and larger stability range for $k_{rc}$.
Frequency Adaptive Periodic Control of power converters:
**Frequency Adaptive Periodic Control of power converters:**

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Nominal value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Grid voltage (RMS) $V_{gn}$</td>
<td>220</td>
<td>V</td>
</tr>
<tr>
<td>Grid frequency $f_0$</td>
<td>50</td>
<td>Hz</td>
</tr>
<tr>
<td>Current reference amplitude $I_g^*$</td>
<td>5</td>
<td>A</td>
</tr>
<tr>
<td>Transformer leakage inductance $L_g$</td>
<td>2</td>
<td>mH</td>
</tr>
<tr>
<td>LCL filter inductor $L_1$ and $L_2$</td>
<td>3.6</td>
<td>mH</td>
</tr>
<tr>
<td>LCL filter capacitor $C_f$</td>
<td>2.35</td>
<td>µF</td>
</tr>
<tr>
<td>DC bus voltage $v_{dc}$</td>
<td>400</td>
<td>V</td>
</tr>
<tr>
<td>Switching frequency</td>
<td>10</td>
<td>kHz</td>
</tr>
<tr>
<td>Sampling frequency</td>
<td>10</td>
<td>kHz</td>
</tr>
<tr>
<td>Repetitive control gain $k_{rc}$</td>
<td>1.8</td>
<td>-</td>
</tr>
</tbody>
</table>
Frequency Adaptive Periodic Control of power converters:

\[
v_{\text{inv}}^* (k) = \frac{1}{v_{\text{dc}}(k)} \left[ v_g(k) + b_1 i_g^*(k) - (b_1 - b_2) i_g(k) \right]
\]
Application Case – Results

Deadbeat Control of the grid-connected single-phase converter: without any periodic control

![Graph showing voltage and current waveforms with harmonic analysis.]

- THD $i_g = 8\%$
- 3rd harmonic
- 5th harmonic
- 7th harmonic

<table>
<thead>
<tr>
<th>Frequency</th>
<th>Value</th>
<th>Mean</th>
<th>Min</th>
<th>Max</th>
<th>Std Dev</th>
</tr>
</thead>
<tbody>
<tr>
<td>50.01 Hz</td>
<td>50.01</td>
<td>50.01</td>
<td>50.01</td>
<td>50.01</td>
<td>0.000</td>
</tr>
</tbody>
</table>
Deadbeat Control of the grid-connected single-phase converter: without any periodic control

\[ h_r(j) = \frac{\sum_{j=1}^{199} M_i}{\sum_{j=1}^{199} M_i} \times 100\% \]

\[ Q(z) = 0.1z + 0.8 + 0.1z^{-1} \]

\[ f_{cutoff} \approx 3.5 \text{ kHz} \]

\[ G_{OHC}(z) = \sum_{m \in N_m} G_{sm}(z) = G_{40}(z) + G_{41}(z) + G_{42}(z) \]

\[ k_0 = 0.2, \; k_1 = 1.4, \; k_2 = 0.2 \]
Deadbeat Control of the grid-connected single-phase converter:
with various periodic control
Deadbeat Control of the grid-connected single-phase converter: with various periodic control
Application Case – Results

**Deadbeat Control of the grid-connected single-phase converter:**

with various **periodic control**

![Graph showing THD against grid frequency for different control methods: Deadbeat (DB), Classic RC, FA-OHC, FA-CRC. The graph illustrates the performance of these control methods across varying grid frequencies.](image-url)
Deadbeat Control of the grid-connected single-phase converter:
with various periodic control

Convergence rate of FA-OHC is up to $n/2$ times faster than that of FA-RC.
This is not affected by the frequency adaptive scheme.
Summary

Frequency Adaptive Periodic Control:

Lagrange interpolation FIR FD filter based Frequency Adaptive Periodic Control (FAPC) at a fixed sampling rate,

- FIR FD filter is **always stable**
- It achieves **fast on-line tuning** of the fractional delay and **fast update** of the coefficients
- It offers a **simple but very accurate** real-time frequency adaptive control solution
- Design of the FAPC is **compatible** with non-frequency-adaptive PC systems
Questions?
Part 3

Further Exploration
Beyond periodic signal control:

We are not just controlling periodic signals.
**Digital Multi-Period RC system:**

\[
G_R(z) = \sum_{j=1}^{p} R_j(z) + \sum_{j,k=1}^{p} R_j(z) R_k(z) H(z) + \prod_{j=1}^{p} R_j(z) H^2(z)
\]

\[
R_j(z) = k_j \frac{Q(z) z^{-N_j}}{1 - Q(z) z^{-N_j}} G_f(z)
\]
Digital Multi-Period RC system:

\[
G_R(z) = \sum_{j=1}^{p} R_j(z) + \sum_{j,k=1}^{p} R_j(z) R_k(z) H(z) + \prod_{j=1}^{p} R_j(z) H^2(z)
\]
Digital Multi-Period RC system:

Stability Conditions:

- Roots of $1 + G_c(z)G_p(z) = 0$ are inside the unit circle, i.e., $H(z)$ is stable.
- Roots of $1 - (1 - k_j G_f(z)H(z))Q(z)z^{-N_j} = 0$ are inside the unit circle.

$$\left|1 - k_j G_f(z)H(z)\right|Q(z) < 1$$
Multi-Period Resonant Control:

\[ G_R(z) = \sum_{j=1}^{p} R_j(z) = \sum_{j=1}^{p} k_j \cdot \frac{Q(z)z^{-N_j}}{1 - Q(z)z^{-N_j}} G_f(z) \]
Multi-Period Resonant Control:

\[
G_{MR}(s) = \sum_{j=1}^{p} R_{Mj}(s) = \sum_{j=1}^{p} \left( \sum_{h \in N_{jh}} R_{jh}(s) \right)
\]

\[
R_{Mj}(s) = \sum_{h \in N_{jh}} R_{jh}(s) = \sum_{h \in N_{jh}} \left( k_{hj} \frac{s \cos \theta_{hj} - \omega_{hj} \sin \theta_{hj}}{s^2 + \omega_{hj}^2} \right)
\]
Enhancing the Control by filtering periodic harmonics:

If the feedback controller $G_c(z) \to \infty$, then $y(z) \to r(z)$, even in the presence of disturbances $d(z)$ in the system.

However, the reference $r(z)$ may suffer from unexpected harmonics and leads to harmonics and distort output signals, which feedback controller $G_c(z)$ cannot handle it.

It calls for Periodic Signal Filtering.
**Links** between notch filters and resonant controllers:

A **periodic signal filter** should be able to attenuate the harmonic at a specific frequency to a very low level, meaning that its magnitude response should be low enough.

\[
g_{\text{rsc}}^h (s) = \frac{k_h s}{s^2 + \omega_h^2}
\]
Links between notch filters and resonant controllers:

\[
G_{rsc}^1(s) = \frac{s}{s^2 + (100\pi)^2}
\]
**Links** between notch filters and resonant controllers:

\[
G_h^{\text{notch}}(s) = \frac{1}{1 + G_h^{\text{rsc}}(s)} = \frac{s^2 + \omega_h^2}{s^2 + k_h s + \omega_h^2}
\]

\[
\left| G_h^{\text{notch}}(s) \right|_{s=j\omega_0} = 0
\]
Links between notch filters and resonant controllers:
**Links** between notch filters and resonant controllers:

When considering multiple resonant controllers, a **selective periodic signal filter** (i.e., with **multiple notch frequencies**) can be obtained in the same manner.

\[
G'_{psf}(s) = \frac{1}{1 + \sum_{h=1}^{H} G^h_{rsc}(s)}
\]
**Links** between comb filters and periodic controllers:

Furthermore, as the **conventional RC** can compensate all harmonics, a **full comb filter** can be obtained by including the RC scheme. This should enable filtering out all signals in the frequency range.

\[
G_{psf}(s) = \frac{1}{1 + G_{rc}(s)} = \frac{1}{1 + \frac{e^{-2\pi s/\omega_0}}{1 - e^{-2\pi s/\omega_0}}} = 1 - e^{-2\pi s/\omega_0}
\]

\[
\left|G_{psf}(s)\right|_{s=j\omega_0} \rightarrow 0
\]
**Links** between comb filters and periodic controllers:
Links between comb filters and periodic controllers:

One more step further, what if we consider the selective harmonic control scheme, a unified periodic signal filter for selective periodic signals is obtained.

\[
G_{sc}(s) = \frac{1}{1+G_{sm}(s)} = \frac{e^{-2sT_0/n} - 2\cos\left(\frac{2\pi m}{n}\right)e^{-sT_0/n} + 1}{-\cos\left(\frac{2\pi m}{n}\right)e^{-sT_0/n} + 1}
\]

\[
\left|G_{sm}(s)\right|_{s=j(nk\pm m)\omega_0} \rightarrow 0
\]
Links between comb filters and periodic controllers:

One more step further, what if we consider the selective harmonic control scheme, a unified periodic signal filter for selective periodic signals is obtained.

\[
G_{sc}(z) = \frac{a^2z^{-2N/n} - 2a\cos(2\pi m/n)z^{-N/n} + 1}{-a\cos(2\pi m/n)z^{-N/n} + 1}
\]

\[
G_{esc}(z) = 1 - az^{-N/2}
\]
Links between comb filters and periodic controllers:
Lagrange Polynomial or Virtual Unit Delay

enhancing the frequency adaptability:

\[ Z^{-N_F} = Z^{-N_i} \cdot F = Z^{-N_i} \cdot Z^{-F} \]

Integer part
Easy to implement

Fractional part
Polynomial approximation

\[ Z^{-F} \approx \sum_{k=0}^{n} A_k z^{-k} \quad \text{with} \quad A_k = \prod_{i=0 \atop i \neq k}^{n} \frac{F - i}{k - i} \]

\[ Z^{-N_F} \approx Z^{-N_i} \cdot \sum_{k=0}^{n} A_k z^{-k} \]

Integer part
Easy to implement

Approximated part
Easy to implement
Lagrange Polynomial or Virtual Unit Delay

enhancing the frequency adaptability:

\[ \frac{1}{1 - N_F} = Z^{-N} \cdot Z^{-F} = Z^{-N(1+F_N)} \]

Integer part

Easy to implement

\[ Z_v^{-1} = Z^{-(1+F_N)} \]

Fractional part

\[ Z^{-N_F} \approx Z_v^{-N} \]
Lagrange Polynomial or Virtual Unit Delay

enhancing the frequency adaptability:

![Diagram of Frequency Adaptive Periodic Signal Filters](image)
Periodic Signal Filter to enhance grid synchronization:
**Periodic Signal Filter** to enhance grid synchronization:

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<th>Parameters</th>
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<td>Grid voltage (RMS) $V_{gn}$</td>
<td>230</td>
<td>V</td>
</tr>
<tr>
<td>Grid frequency $f_0$</td>
<td>50</td>
<td>Hz</td>
</tr>
<tr>
<td>LCL filter inverter-side inductor $L_1$</td>
<td>3.6</td>
<td>mH</td>
</tr>
<tr>
<td>LCL filter grid-side inductor $L_2$</td>
<td>4</td>
<td>mH</td>
</tr>
<tr>
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<td>Sampling frequency</td>
<td>10</td>
<td>kHz</td>
</tr>
</tbody>
</table>
Application Case – Results

Periodic Signal Filter to enhance grid synchronization:

- Conventional $T/4$ PLL
- Virtual Unit Delay based $T/4$ PLL
- Virtual Unit Delay based $T/0$ PLL with a Comb Filter

\[ f = 52 \text{ Hz} \quad \text{THD}_V = 14.5\% \]
**Application Case – Results**

**Periodic Signal Filter** to enhance grid synchronization:

THD_{v} = 3.3%
Periodic Signal Filter to enhance the current control:
**Application Case – Results**

**Periodic Signal Filter** to enhance the current control:

![Graph showing comparison between W/O Notch Filter and With Notch Filter](image)

- **W/O Notch Filter**
  - Voltage $v_g$ and current $i_g$ shifted.

- **With Notch Filter**
  - Voltage $v_g$ and current $i_g$ aligned.

Time [5 ms/div]
Application Case – Results

Periodic Signal Filter to enhance the current control:

![Graph showing periodic signal filter results]

- fundamental
- With the notch filter
- W/O the notch filter
Conclusion and Discussion

A Generalized P'T'D Control

combines feedback control and Periodic Control

It will provide a simple but effective general optimal (accuracy, fast, robust, and easy implementation) control solution to periodic signal compensation in extensive engineering applications.
Questions?
Thank you!

Y. Yang, Y. Tang
@ Kaohsiung